

# Learning Models Robust To Adversarial Attacks

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# Outline

- Introduction
- Generating Adversarial Examples
  - FGSM
  - BIM
  - ILCM
- Adversarial Robustness
- Results
- Conclusion

# Introduction

# Can Neural Networks be fooled?



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“Panda”  
57.7% confidence



“Gibbon”  
99.3% confidence

# Can Neural Networks be fooled?



“Panda”  
57.7% confidence

+0.007 \*



“Nematode”  
8.2% confidence

=



“Gibbon”  
99.3% confidence

Figure- Example of an adversarial image for GoogLeNet trained on ImageNet[1].

[1] Goodfellow, Ian J., Jonathon Shlens, and Christian Szegedy. "Explaining and harnessing adversarial examples." *arXiv preprint arXiv:1412.6572* (2014).

# Notations

- $F$  : Learned model/classifier
- $\theta$  : Model/classifier parameters
- $(x, y)$  : The natural image and its true label
- $x'$  : The adversarial image
- $L(y_p, y)$  : The loss function e.g. cross-entropy loss
- $\epsilon$  : The allowed perturbation in the image  $x$

## Formal Definition:

For an image  $x$ ,  $x'$  is termed as its adversarial image if:

- $F(x) \neq F(x')$
- $d(x, x') \leq \epsilon$

where  $d(x, x') = \|x, x'\|_p$  for  $p = \{0, 2, \infty\}$



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## Generating an adversarial image

$$\max_{\delta \in \Delta} L(F(x + \delta), y)$$

where  $\Delta = \{ \delta : \| \delta \|_{\infty} \leq \epsilon \}$

# Toy Example: Binary Linear Classifier<sup>[4]</sup>

For the dataset  $(x, y)$  such that  $x \subseteq \mathbb{R}^d$  and  $\exists y = \{-1, 1\}$ , let  $F$  be the model defined as :

- $F(x) = w^T x + b$
- $p(y = +1 | x) = 1/(1 + \exp(-F(x)))$
- $p(y = -1 | x) = 1/(1 + \exp(F(x)))$

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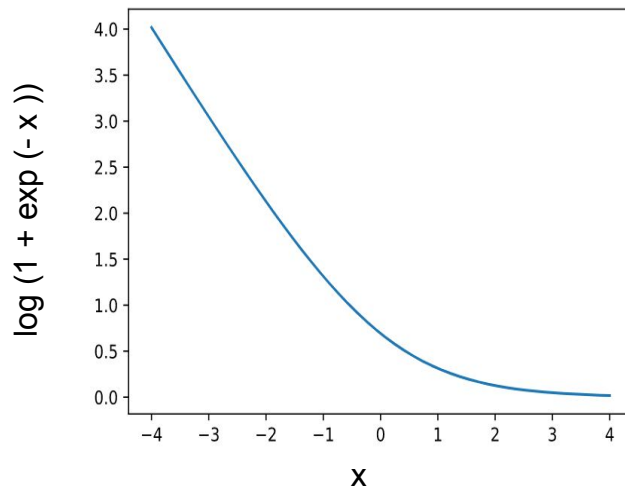
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And  $L$  be the negative log likelihood:

- $L(F(x), y) = \log(1 + \exp(-yF(x)))$

# Adversarial Examples for Binary Linear Classifier[4]

$$L(F(x), y) = \log (1 + \exp (- y F(x) ))$$

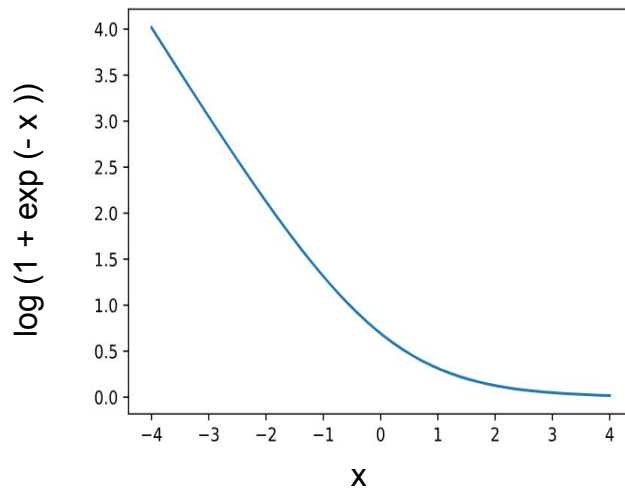


# Adversarial Examples for Binary Linear Classifier[4]

$$L(F(x), y) = \log (1 + \exp (- y F(x) ))$$

$$\max_{\delta} L(F(x + \delta), y)$$

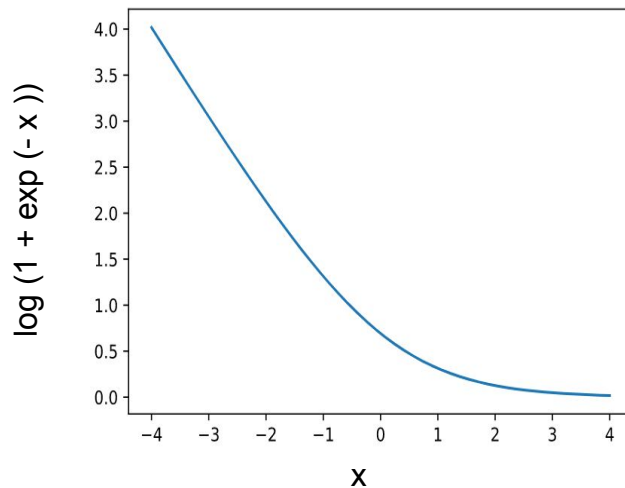
$$= \max_{\delta} \log (1 + \exp (- y F(x + \delta) ))$$



# Adversarial Examples for Binary Linear Classifier[4]

$$L(F(x), y) = \log (1 + \exp (- y F(x) ))$$

$$\begin{aligned} & \max_{\delta} L(F(x + \delta), y) \\ &= \max_{\delta} \log (1 + \exp (- y F(x + \delta) )) \\ &= \min_{\delta} (y F(x + \delta)) \\ &= \min_{\delta} (y(w^T x + b) + y w^T \delta) \\ &= \min_{\delta} (y w^T \delta) \end{aligned}$$

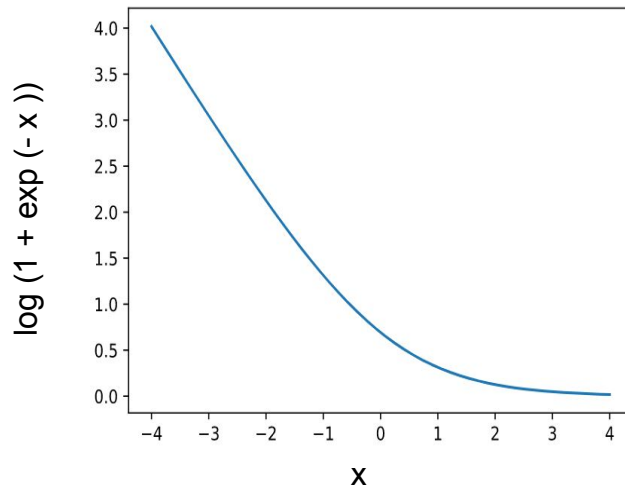


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**For  $L_{\infty}$  norm,  $\delta^* = -y \epsilon \text{sign}(w)$**



# Generating Adversarial Examples



# Generating Adversarial Examples

- Fast Gradient Sign Method (FGSM)
- Basic Iterative Method (BIM)
- Iterative Least-likely Class Method (ILCM)

# 1. Fast Gradient Sign Method (FGSM)[1]

For any model  $F$  and natural image  $x$ , the adversarial image is computed as :

$$x' = x + \epsilon \text{sign} (\nabla_x L( F(x), y ))$$

- It is an  $L_\infty$  attack as  $\|x'-x\|_\infty \leq \epsilon$

[1] Goodfellow, Ian J., Jonathon Shlens, and Christian Szegedy. "Explaining and harnessing adversarial examples." *arXiv preprint arXiv:1412.6572* (2014).

[2] Kurakin, Alexey, Ian Goodfellow, and Samy Bengio. "Adversarial examples in the physical world." *arXiv preprint arXiv:1607.02533* (2016).

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# 2. Basic Iterative Method (BIM)[2]

Repeat FGSM using small step size for  $k$  iterations

- $x'_0 = x$
- $x'_{i+1} = \text{clip}\{ (x'_i + \alpha \text{sign} (\nabla_x L( F(x'_i), y ) )), x + \epsilon, x - \epsilon \}$

[1] Goodfellow, Ian J., Jonathon Shlens, and Christian Szegedy. "Explaining and harnessing adversarial examples." *arXiv preprint arXiv:1412.6572* (2014).

[2] Kurakin, Alexey, Ian Goodfellow, and Samy Bengio. "Adversarial examples in the physical world." *arXiv preprint arXiv:1607.02533* (2016).

### 3. Iterative Least-Likely Class Method (ILCM)[2]

- The least likely class  $y_{LL}$  is given as :

$$y_{LL} = \arg \min_y p(y | x)$$

- For  $y_{LL}$  to be the target label of the adversarial image,

- $y_{LL} = \arg \max_y p(y | x')$
- $L(F(x'), y_{LL})$  should be minimised

- Adversarial image  $x'$  s.t.  $\|x' - x\|_\infty \leq \epsilon$  is computed as :

- $x'_0 = x$
- $x'_{i+1} = \text{clip}\{ (x'_i - \alpha \text{sign}(\nabla_x L(F(x'_i), y_{LL}))), x + \epsilon, x - \epsilon \}$

# Examples of Adversarial Images



Clean Image



Fast Gradient Sign  
Method (  $L_\infty = 32$  )



Basic Iterative  
Method (  $L_\infty = 32$  )



Iterative Least-likely  
Class Method (  $L_\infty = 28$  )

Figure : Generating adversarial images using different attacks for  $\epsilon = 32$  [2].

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Figure : Generating adversarial images using different attacks for  $\epsilon = 32$  [2].

**Iterative methods result in finer perturbations in comparison to the fast method**

# Which attack is better?

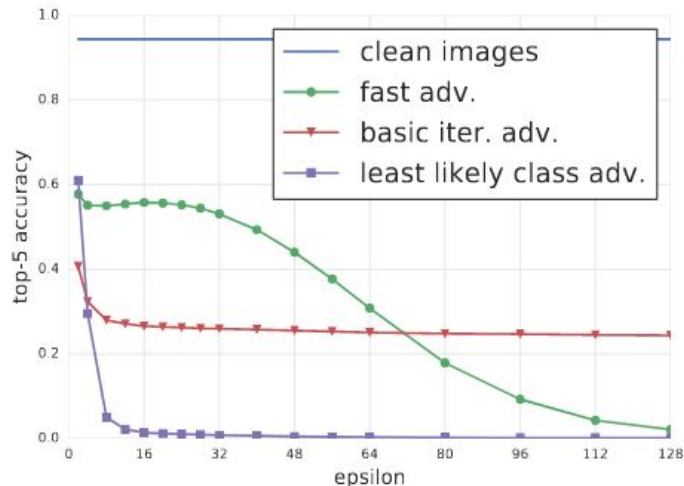
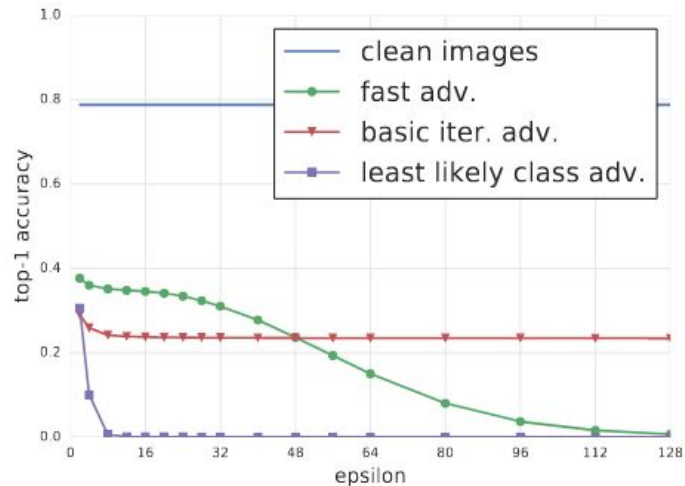


Figure - Drop in accuracy wrt different attacks on Inception v3 network trained on ImageNet dataset [2]

[2] Kurakin, Alexey, Ian Goodfellow, and Samy Bengio. "Adversarial examples in the physical world." *arXiv preprint arXiv:1607.02533* (2016).

# Robustness against Adversarial Examples



# Adversarial Training as a Robust Optimisation Problem[3]

- For a dataset  $\mathcal{D}$  and allowed set of perturbations  $\mathcal{S}$ , a robust model can be trained by minimising the following optimisation :

$$\min_{\theta} \rho(\theta), \text{ where } \rho(\theta) = \mathbb{E}_{(x_i, y_i) \sim \mathcal{D}} \left[ \max_{\|\delta\|_p \leq \epsilon} L(\theta, x_i + \delta, y_i) \right]$$

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Adversarial Images  
Loss

- Solving the inner-optimisation problem using Basic Iterative Method(BIM) also known as Projected Gradient Descent (PGD)

# Results of Adversarial Training

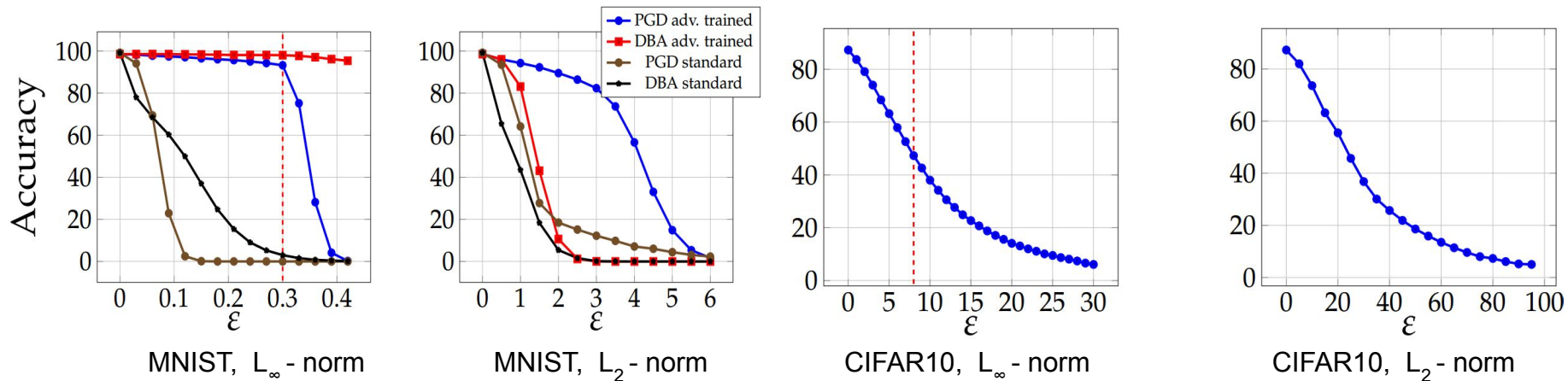


Figure - Robustness of adversarially trained networks against PGD adversaries of different strength. The models are trained on generated PGD adversarial images using  $\epsilon=0.3$  and  $\epsilon=8$  for MNIST and CIFAR10 respectively [3].

[3] Madry, Aleksander, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. "Towards deep learning models resistant to adversarial attacks." *arXiv preprint arXiv:1706.06083* (2017).

# Does Increasing Network Capacity Help?

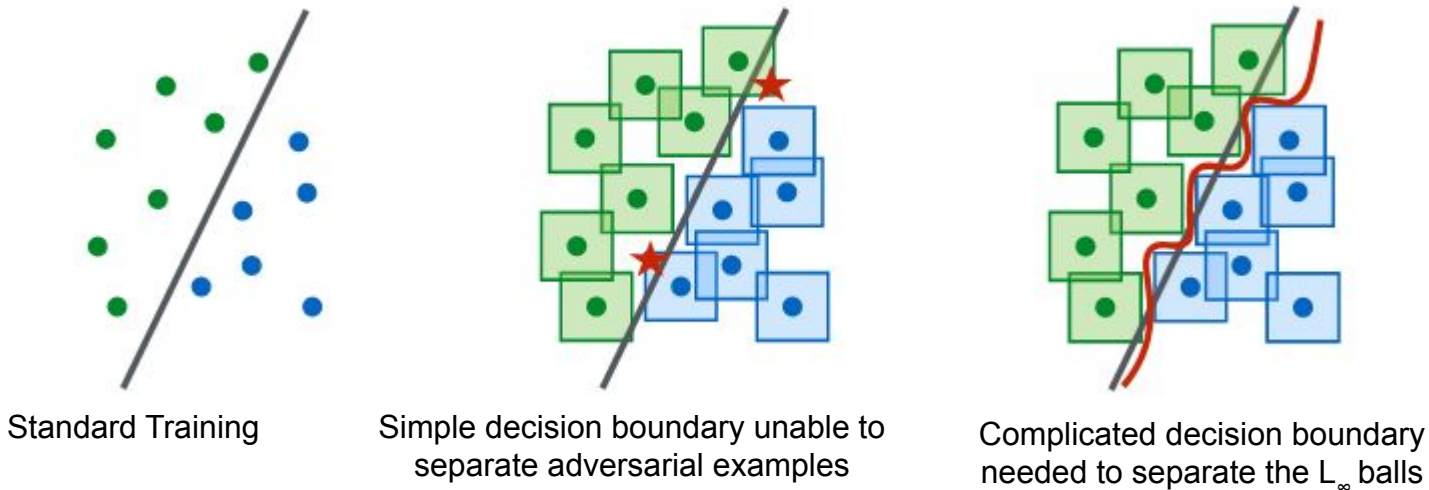


Figure - A conceptual illustration of standard vs. adversarial decision boundaries[3].

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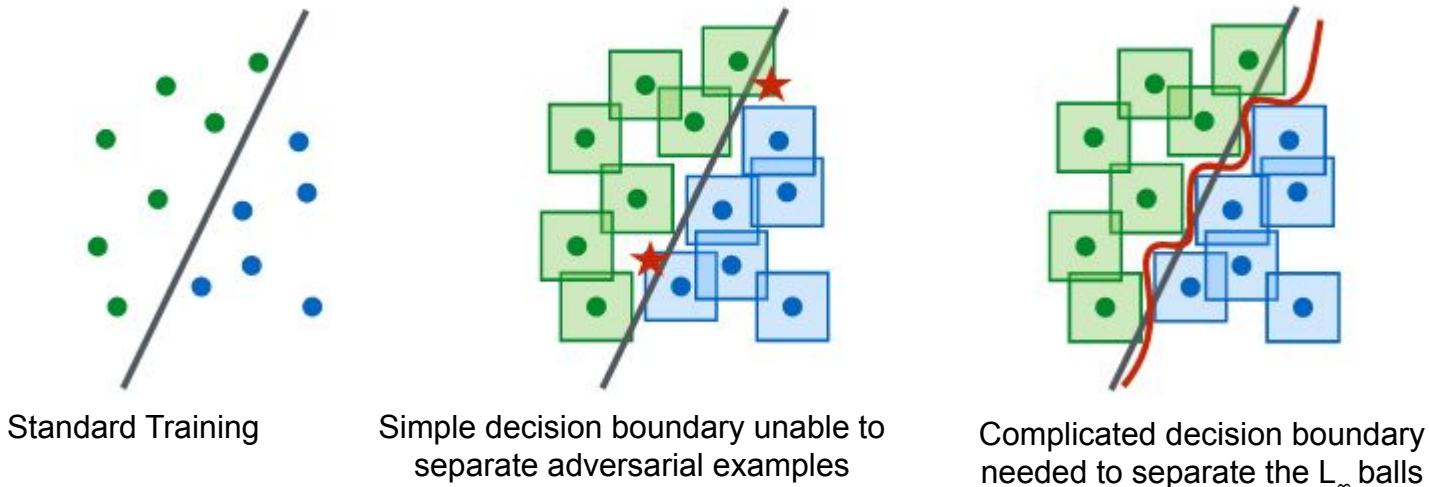


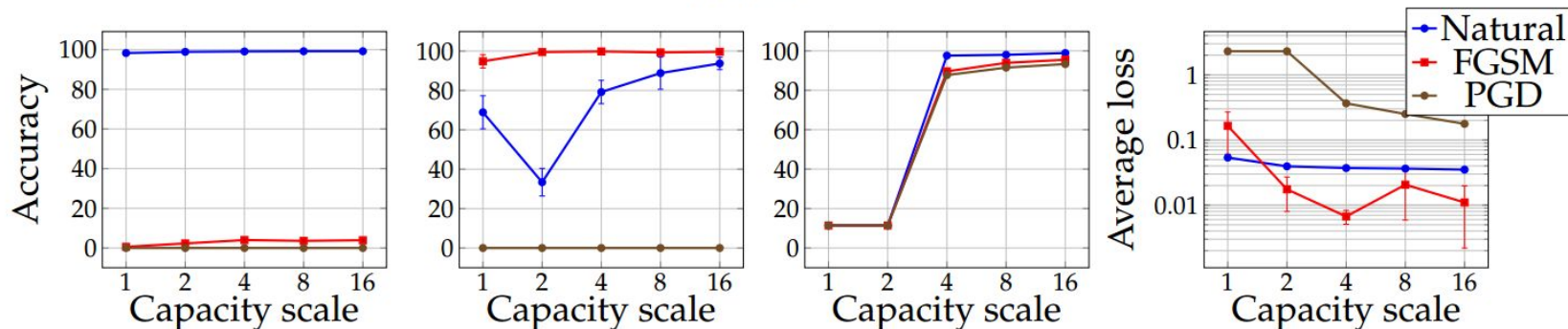
Figure - A conceptual illustration of standard vs. adversarial decision boundaries[3].

**Increasing network capacity does help in improving the adversarial robustness of the model**

[3] Madry, Aleksander, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. "Towards deep learning models resistant to adversarial attacks." *arXiv preprint arXiv:1706.06083* (2017).

# Results of Increasing Network Capacity

MNIST



CIFAR10

	Simple	Wide	Simple	Wide	Simple	Wide	Simple	Wide
(a) Standard training	92.7%	95.2%	87.4%	90.3%	79.4%	87.3%	0.00357	0.00371
(b) FGSM training	27.5%	32.7%	90.9%	95.1%	51.7%	56.1%	0.0115	0.00557
(c) PGD training	0.8%	3.5%	0.0%	0.0%	43.7%	45.8%	1.11	0.0218

Figure - The adversarial robustness of the model improves with increasing network capacity[3].

[3] Madry, Aleksander, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. "Towards deep learning models resistant to adversarial attacks." *arXiv preprint arXiv:1706.06083* (2017).

# Conclusion

- Adversarial images can be generated easily
- Adversarial training helps in improving the robustness but it starts failing for  $\epsilon$  greater than training  $\epsilon$

# References

1. Goodfellow, Ian J., Jonathon Shlens, and Christian Szegedy. "Explaining and harnessing adversarial examples." *arXiv preprint arXiv:1412.6572* (2014).
2. Kurakin, Alexey, Ian Goodfellow, and Samy Bengio. "Adversarial examples in the physical world." *arXiv preprint arXiv:1607.02533* (2016).
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4. [https://adversarial-ml-tutorial.org/linear\\_models](https://adversarial-ml-tutorial.org/linear_models)

Thank You